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Engineering Notes

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Attitude Acquisition of a Satellite with a Partially Filled Liquid Tank

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Introduction

ACQUISITION techniques by angular momentum exchange have been studied for a long time by many researchers [1–9]. In this paper, we consider the reorientation of a bias momentum stabilized satellite that is equipped with a momentum wheel and spherical fuel tank. Even though many studies regarding the momentum transfer problem have been done, it appears that none have dealt with it in combination with liquid sloshing. The objective of the present study, then, is to investigate the effect of liquid sloshing on attitude acquisition using a momentum transfer maneuver. In particular, the study focuses on how the viscosity of the liquid and the location of the center of the tank influence the attitude acquisition of the spacecraft. As an equivalent model representing the slosh motion of a liquid, a spherical pendulum is adopted [10,11]. Also, equations of motion for the system composed of three bodies are derived. Results from a digital simulation based on the equations of motion are presented. The spacecraft is assumed to be initially spin stabilized about the axis of the maximum moment of inertia and the wheel spin axis is parallel to the axis of the spacecraft's intermediate moment of inertia.

System Model and Equations of Motion

Figure 1 shows a typical momentum biased satellite system consisting of the main body $\mathbf{B}$ of a spacecraft, a momentum wheel $\mathbf{W}$, and a liquid fuel tank $\mathbf{F}$. The main body has mass m_b and inertia dyadic $\mathbf{I}_b$ about the x, y, and z axes. The momentum wheel has mass m_w and inertia dyadic $\mathbf{I}_w$ about the x, y, and z axes. The wheel spin axis is fixed in the body frame and aligned with the z axis. The integrated body $\mathbf{B_I}$ (not shown in the figure) consisting of the main body $\mathbf{B}$ and the momentum wheel $\mathbf{W}$ has mass m_1 and center of mass C_1, where m_1 is the sum of m_b and m_w. The pendulum has center of mass C_2 and consists of a point mass m_2 attached to a rigid, massless rod of length r, which in turn is attached at a point O to the body $\mathbf{B}$. Its motion with respect to the body $\mathbf{B}$ is defined by the angles θ and ψ.

Equations of motion may be derived for the system of three bodies by using various methods. In the present study, the Newton–Euler method is used and we assume that the center of the momentum wheel is located at C_1. The x, y, and z axes are fixed in the reference body $\mathbf{B}$ with the origin at C_1 and rotate with $\mathbf{B}$ at an angular velocity $\boldsymbol{\omega}$. If we define the position vector from the system mass center C to C_1 as $\mathbf{r}_1$, that from C to C_2 as $\mathbf{r}_2$, and the angular velocity vector of the wheel relative to $\mathbf{B}$ as Ω, then the equations governing the attitude motion of the system may be derived by first writing the angular momentum of the system about C in the form

$$\mathbf{H} = \mathbf{I}_b \cdot \boldsymbol{\omega} + \mathbf{I}_w \cdot (\boldsymbol{\omega} + \Omega) + \mu m_1 \mathbf{r}_p \times (\boldsymbol{\omega} \times \mathbf{r}_p + \dot{\mathbf{r}}_p) \tag{1}$$

where $\mathbf{r}_p = \mathbf{r}_o + \mathbf{r}$, $\mathbf{r} = r(\sin\theta\cos\psi \;\; \sin\theta\sin\psi \;\; -\cos\theta)^T$, in which $\mathbf{r}_o$ is a position vector from C_1 to the center of the tank O, $\dot{\mathbf{r}}_p$ is the time rate of change of $\mathbf{r}_p$ due to the motion of the pendulum relative to $\mathbf{B}$, and $\mu = m_2/(m_1 + m_2)$. By writing Eq. (1) in matrix form, we have

$$\mathbf{H} = \mathbf{J}\boldsymbol{\omega} + \mathbf{h}_w + \mathbf{h}_p \tag{2}$$

where $\mathbf{J} = \mathbf{I}_b + \mathbf{I}_w - \mu m_1 \mathbf{r}_p^\times \mathbf{r}_p^\times$, $\mathbf{h}_w = \mathbf{I}_w \Omega$, and $\mathbf{h}_p = \mu m_1 \mathbf{r}_p^\times \dot{\mathbf{r}}_p$. Here, the cross superscript $\times$ over $\mathbf{r}_p$ denotes the skew-symmetric matrix used to form components of cross products of vectors [12]. The time derivative of the vector $\mathbf{H}$ and Eq. (2) may be used to obtain the matrix equations,

$$\dot{\boldsymbol{\omega}} = \mathbf{J}^{-1}[\{\mathbf{H}^\times + \mu m_1 r^2(\dot{\mathbf{u}}_r^\times \mathbf{u}_p^\times + \mathbf{u}_p^\times \dot{\mathbf{u}}_r^\times)\}\boldsymbol{\omega} - \boldsymbol{\alpha} - \mu m_1 r^2 \mathbf{u}_p^\times \ddot{\mathbf{u}}_r^\times] \tag{3}$$

$$\dot{\mathbf{h}}_w = \boldsymbol{\alpha} \tag{4}$$

where $\mathbf{u}_r = \mathbf{r}/r$ and $\mathbf{u}_p = \mathbf{r}_p/r$.

An equation for the relative motion of the pendulum is more difficult to find. One way to obtain a suitable equation is to write the absolute acceleration of C_2, equate the product of m_2 and this acceleration to the force on C_2, and cross $\mathbf{r}$ into both sides of the resulting equation. Under the assumption that C is stationary in space, the absolute acceleration of C_2 can be expressed as

$$\begin{aligned}\mathbf{a}_p &= \frac{\mathrm{D}^2\mathbf{r}_2}{\mathrm{D}t^2} = (1-\mu)\frac{\mathrm{D}^2\mathbf{r}_p}{\mathrm{D}t^2} \\ &= (1-\mu)[(\ddot{\mathbf{r}} + \dot{\boldsymbol{\omega}}^\times \mathbf{r}_p + 2\boldsymbol{\omega}^\times \dot{\mathbf{r}} + \boldsymbol{\omega}^\times(\boldsymbol{\omega}^\times \mathbf{r}_p)]\end{aligned} \tag{5}$$

where $\mathbf{r}_2$ is the position vector from C to C_2 and $\mathrm{D}(\cdot)/\mathrm{D}t$ is the time derivative of a vector as seen from a nonrotating reference frame. Therefore, the moment equation about the pendulum pivot point O becomes

$$\mu m_1 \mathbf{r}^\times(\ddot{\mathbf{r}} - \mathbf{r}_p^\times \dot{\boldsymbol{\omega}} + 2\boldsymbol{\omega}^\times \dot{\mathbf{r}} - \boldsymbol{\omega}^\times \mathbf{r}_p^\times \boldsymbol{\omega}) = \mathbf{T}_o \tag{6}$$

$\mathbf{T}_o$ may be replaced by a simple viscous torque model such as

$$\mathbf{T}_o = (-\beta_\theta \dot{\theta} \sin\psi \quad \beta_\theta \dot{\theta}\cos\psi \quad -\beta_\psi \dot{\psi} \sin^2\psi)^T \tag{7}$$

where $\beta_\theta = C_\theta r^2$ and $\beta_\psi = C_\psi r^2$. If $\dot{\boldsymbol{\omega}}$ is eliminated from Eq. (6), the resulting equation becomes

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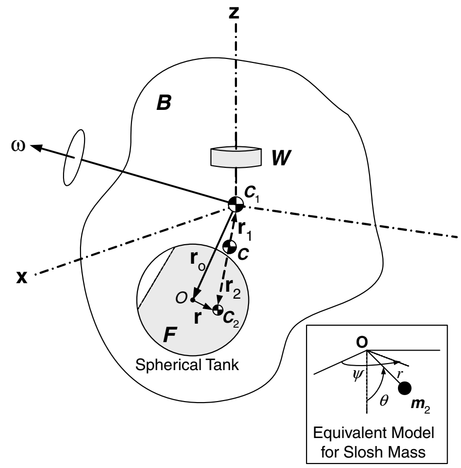


Fig. 1 Three-body system model.

$$\mu m_1 r^2 \mathbf{u}_r^\times[(\mathbf{E} + \mu m_1 r^2 \mathbf{u}_p^\times \mathbf{J}^{-1}\mathbf{u}_p^\times)\ddot{\mathbf{u}}_r - \mathbf{u}_p^\times \mathbf{J}^{-1}\{(\mathbf{H}^\times - \dot{\mathbf{J}})\boldsymbol{\omega} - \boldsymbol{\alpha}\} + 2\boldsymbol{\omega}^\times \dot{\mathbf{u}}_r - \boldsymbol{\omega}^\times \mathbf{u}_p^\times \boldsymbol{\omega}] = \mathbf{T}_o \tag{8}$$

where $\mathbf{E}$ is an identity matrix. Then, we may find the necessary equations by using the matrix

$$\mathbf{C} = \begin{bmatrix} \sin\psi & -\cos\psi & 0 \\ \cos\psi & \sin\psi & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{9}$$

The first row of $\mathbf{C}$ is a unit vector perpendicular to the plane in which θ is measured. The second row is a unit vector orthogonal to the first row. Let $\boldsymbol{\xi} = [\theta \quad \psi]^T$. Then, since

$$\dot{\mathbf{u}}_r = \mathbf{A}_2 \dot{\boldsymbol{\xi}} \tag{10}$$

the equation of motion for the pendulum can be written as follows:

$$\mathbf{C}\mathbf{A}_1 \mathbf{A}_2 \ddot{\boldsymbol{\xi}} = \mathbf{C}(\mathbf{A}_3 - \mathbf{A}_1 \dot{\mathbf{A}}_2 \dot{\boldsymbol{\xi}}) \tag{11}$$

where

$$\mathbf{A}_1 = \mathbf{u}_r^\times(\mathbf{E} + \mu m_1 r^2 \mathbf{u}_p^\times \mathbf{J}^{-1}\mathbf{u}_p^\times) \tag{12}$$

$$\mathbf{A}_2 = \begin{bmatrix} \cos\theta\cos\psi & -\sin\theta\sin\psi \\ \cos\theta\sin\psi & \sin\theta\cos\psi \\ \sin\theta & 0 \end{bmatrix} \tag{13}$$

$$\mathbf{A}_3 = \mathbf{u}_r^\times[\mathbf{u}_p^\times \mathbf{J}^{-1}(\mathbf{H}^\times - \dot{\mathbf{J}}) + 2\dot{\mathbf{u}}_r^\times + \boldsymbol{\omega}^\times \mathbf{u}_p^\times]\boldsymbol{\omega} - \mathbf{u}_r^\times \mathbf{u}_p^\times \mathbf{J}^{-1}\boldsymbol{\alpha} + \mathbf{T}_o/\mu m_1 r^2 \tag{14}$$

Next, let $\mathbf{D} = \mathbf{C}\mathbf{A}_1\mathbf{A}_2$ and $\mathbf{Q} = (q_1 \quad q_2 \quad q_3)^T = \mathbf{C}(\mathbf{A}_3 - \mathbf{A}_1\dot{\mathbf{A}}_2\dot{\boldsymbol{\xi}})$. We then have

$$\ddot{\theta} = q_1 \tag{15}$$

$$\ddot{\psi} = q_2/(\cos\theta\sin\theta) \tag{16}$$

Numerical Examples

In these examples, the following basic data are used throughout all simulations in this paper:

$$\mathbf{I}_b = \begin{bmatrix} 503 & 0 & 0 \\ 0 & 385 & -5 \\ 0 & -5 & 420 \end{bmatrix} \text{kg}\cdot\text{m}^2, \quad \mathbf{I}_w = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.17 \end{bmatrix} \text{kg}\cdot\text{m}^2$$

$$m_1 = 1000\ \text{kg}, \quad m_2 = 0.1 m_1, \quad r = 15\ \text{cm}$$

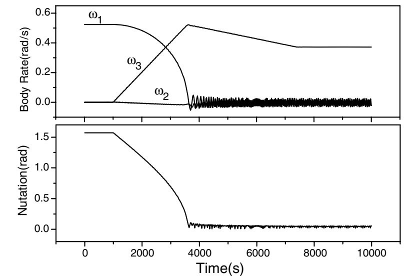
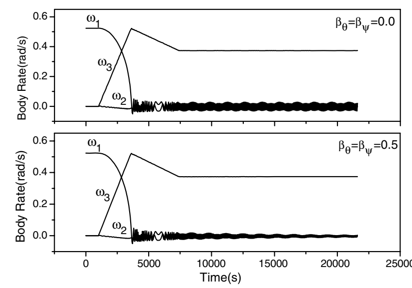
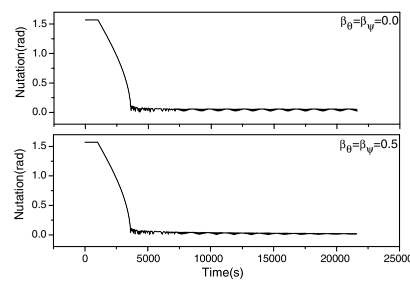


Fig. 2 Angular rate and nutation of the body with an unfilled tank.



a)



b)

Fig. 3 a) Viscosity effect on body rate; b) viscosity effect on nutation.

Initially the spacecraft is spin stabilized about the axis of maximum moment of inertia and the wheel axis is parallel to the axis of intermediate moment of inertia. The attitude acquisition maneuver starts with the configuration of $\boldsymbol{\omega}(0) = (5 \quad 0 \quad 0)^T$ rpm. A motor is then energized to spin the wheel relative to the body. Because of the conservation of angular momentum, the spinup of the wheel transfers momentum from the body to the wheel, thus arriving at a configuration in which the spacecraft's intermediate moment of inertia axis is reoriented parallel to the angular momentum vector. In the simulation, the speed of the wheel remains zero for the period of the first 1000 s and then increases linearly until it reaches 6000 rpm at 7400 s. It then remains constant. The wheel spinup is summarized as follows:

$$\Omega_1 = \Omega_2 = 0, \qquad 0 \le t \le t_f,$$

$$\Omega_3 = \begin{pmatrix} 0 & 0 \le t < 1000 \\ 0.9375(t - 1000) & 1000 \le t < 7400 \\ 6000 & 7400 \le t \le t_f \end{pmatrix} \text{rpm}$$

where Ω_1, Ω_2, and Ω_3 are the x, y, and z components, respectively, of the vector Ω.

Momentum Exchange of a Spacecraft with an Unfilled Tank

This simulation demonstrates the standard reorientation maneuver applied to a conventional two-body model composed of a spacecraft body and a wheel. Because the spacecraft is equipped with an empty fuel tank, the slosh mass and pendulum string length are set to zero (i.e., $m_2 = r = 0$). Figure 2 shows time histories of the angular rate

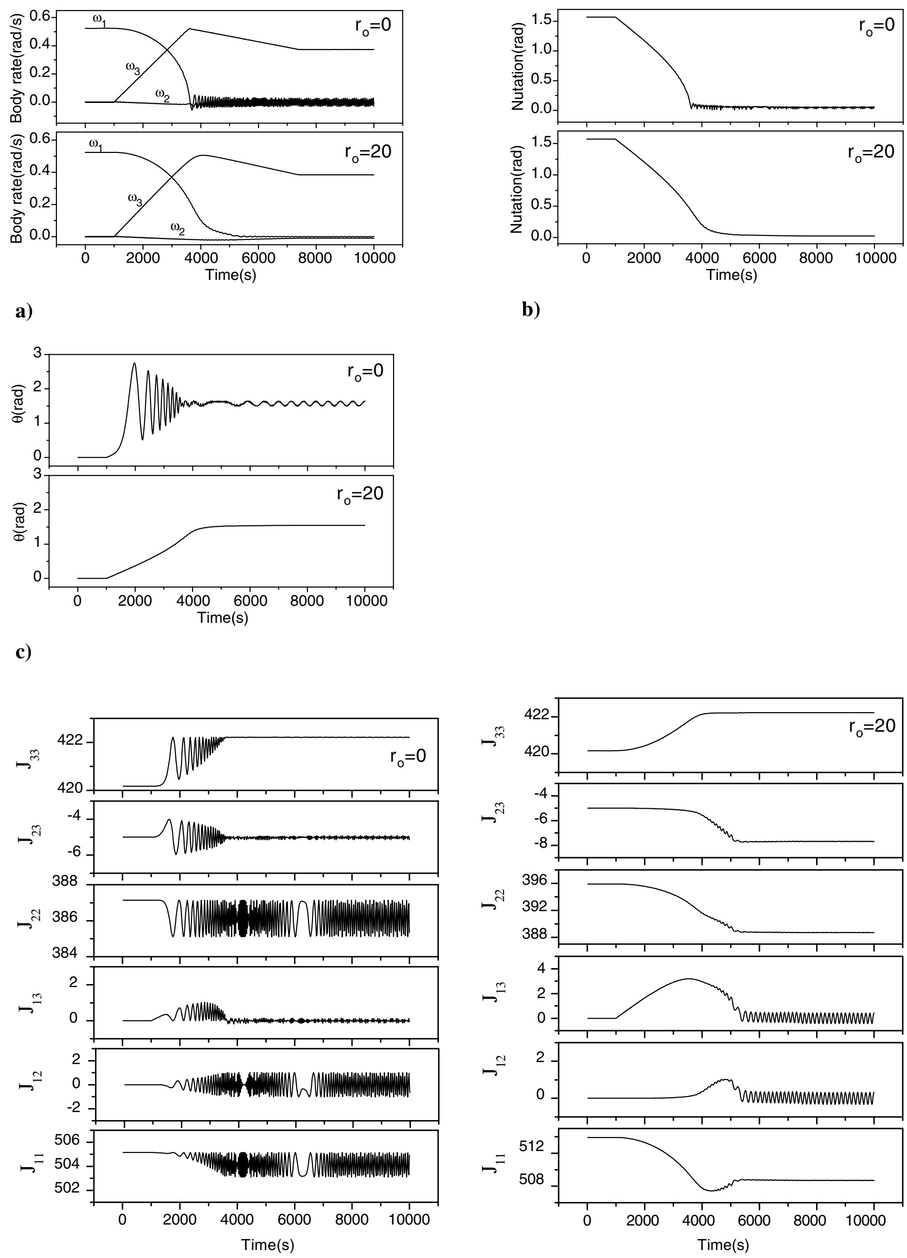


Fig. 4 a) Effect of the tank's center location on body rate; b) effect of the tank's center location on nutation; c) effect of the tank's center location on slosh motion; d) effect of the tank's center location on moment of inertia.

and nutation angle of the spacecraft both during and after the maneuver. The residual nutation varies from 1.3 to 3.2 deg.

Effects of Viscosity of Liquid

We consider an acquisition sequence that begins with a 5 rpm x-axis initial spin rate and lasts for roughly 4.5 h. And we observed the effects of viscosity on the reorientation maneuver for 6 h. In the simulation, the center of the tank is always modeled to be located at the center of mass of the body $\mathbf{B_I}$, that is, $\mathbf{r}_o = (0 \quad 0 \quad 0)^T$. The results demonstrate that the viscosity of the liquid does damp out the nutational motion during and after the maneuver, but requires heavy damping and a long period of time for stabilization. Figures 3a and 3b show time histories for the angular rates and nutation, respectively. In each figure, two examples are given to illustrate the effect of viscosity on the body. The wobbling of the spacecraft in its final state is reduced only when heavy viscous coefficients were used. Residual nutation varies from 1.10 to 3.42 deg when $\beta_\theta = \beta_\psi = 0$, and from 0.03 to 0.93 deg when $\beta_\theta = \beta_\psi = 0.5$.

Effects of Location of Center of Tank

This simulation demonstrates how the location of the center of the tank influences the whole body dynamics of a spacecraft during and after the maneuver. It was assumed that the center of the tank is on the z axis of the body, that is, $\mathbf{r}_o = (0 \quad 0 \quad -r_o)^T$. To observe the effect of the specific location of the center, the liquid in the system was assumed to be inviscid although the other initial conditions used throughout all of the simulations remained the same. The simulation took less than 3 h. Figures 4a–4c show time histories of the angular rates, nutation, and pendulum motion, respectively. In each figure, two representative examples are given to illustrate the effect of the corresponding location of the center of the tank on the body. As can be seen in the figures, the wobbling of the spacecraft is noticeable when the location of the center of the tank is around C_1. However, as

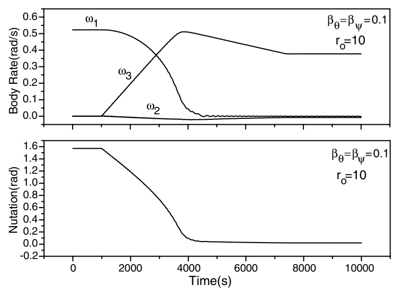


Fig. 5 Combined effects of liquid viscosity and location of the tank.

the location of the center of the tank moves away from C_1, wobbling decreases significantly. Also, the residual nutation angle reduces to 1.44 deg when r_o is 20 cm. As can be observed in Fig. 4c, when the location of the center of the tank is approximately at the center of mass of the body $\mathbf{B_I}$, the force exerted on the slosh mass by the body is relatively small and the slosh mass undergoes large oscillations. On the other hand, when the location of the center of the tank moves away from the center of mass of the body, the centrifugal force increases. Therefore, the slosh mass moves to a new equilibrium point without oscillations. Figure 4d illustrates the time histories of the spacecraft's moment of inertia during and after the maneuver.

Combined Effects of Liquid Viscosity and Location of Tank

Lastly we conducted a simulation to investigate the combined effects of liquid viscosity and the location of the center of the tank on the attitude motion of the spacecraft. Parameter values for the combined slosh model were varied throughout the simulation. However, the combined slosh model and the inviscid slosh model exhibited very similar results. That is, when the center of the tank is some distance away from the center of mass of the body $\mathbf{B_I}$, the attitude motion is almost the same regardless of increases or decreases in the values of the viscous coefficients. One example is given in Fig. 5. A comparison of Figs. 4 and 5 reveals almost identical time histories, implying that the effect of the centrifugal force of the liquid on the spacecraft motion is quite a bit stronger compared to the effect of liquid viscosity.

Conclusions

A study of attitude acquisition by momentum exchange was carried out on a satellite that is equipped with a spherical fuel tank and a momentum wheel fixed in the body frame. Full nonlinear equations of motion of a three-body system that consists of a main spacecraft body, a momentum wheel, and a spherical pendulum representing liquid fuel were derived. Computer simulations were conducted for several cases mainly to observe how fuel viscosity and the location of the center of the tank influence the attitude acquisition of the satellite. Simulation results show that both the viscosity and the location of the center of the tank affect the attitude motion of the spacecraft, but that the location of the center of the tank has a greater effect than the viscosity on stabilization. Damping of the nutational motion takes a long time and requires relatively large coefficients of viscosity. Wobbling of the spacecraft is noticeable when the location of the center of the tank is near the center of mass of the body. As the center of the tank moves away from the center of mass of the body along the wheel spin axis, however, the wobbling decreases significantly.

Acknowledgments

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References

[1] Gebman, J. R., and Mingori, D. L., "Perturbation Solution for the Flat Spin Recovery of a Dual-Spin Spacecraft," *AIAA Journal*, Vol. 14, No. 7, July 1976, pp. 859–867.

[2] Barba, P. M., and Aubrun, J. N., "Satellite Attitude Acquisition by Momentum Transfer," *AIAA Journal*, Vol. 14, No. 10, Oct. 1976, pp. 1382–1386.

[3] Hubert, C., "Spacecraft Attitude Acquisition from an Arbitrary Spinning or Tumbling State," *Journal of Guidance and Control*, Vol. 4, No. 2, March 1981, pp. 164–170.

[4] Vigneron, F. R., and Staley, D. A., "Satellite Attitude Acquisition by Momentum Transfer—The Controlled Wheel Speed Method," *Celestial Mechanics*, Vol. 27, No. 2, June 1982, pp. 111–130. doi:10.1007/BF01271687

[5] Vadali, S. R., and Junkins, J. L., "Spacecraft Large Angle Rotational Maneuvers with Optimal Momentum Transfer," *Journal of Astronautical Sciences*, Vol. 31, No. 2, 1983, pp. 217–235.

[6] Weissberg, J., and Ninomiya, K., "Improved Method for the Initial Attitude Acquisition Maneuver," *Journal of Guidance, Control, and Dynamics*, Vol. 10, No. 3, May 1987, pp. 316–319.

[7] Hall, C. D., and Rand, R. H., "Spinup Dynamics of Axial Dual-Spin Spacecraft," *Journal of Guidance, Control, and Dynamics*, Vol. 17, No. 1, 1994, pp. 30–37.

[8] Oh, H. S., "Spinup Maneuver and Nonlinear Active Nutation Damping," *Journal of Guidance, Control, and Dynamics*, Vol. 19, No. 3, 1996, pp. 704–706.

[9] Wuhr, M., and Staley, D. A., "Satellite Reorientation Maneuvers by Momentum Transfer," AIAA Paper 625-421, 17–21 May 2004.

[10] Kang, J. Y., and Cochran, J. E. Jr., "Resonant Motion of a Spin-Stabilized Thrusting Spacecraft," *Journal of Guidance, Control, and Dynamics*, Vol. 27, No. 3, May 2004, pp. 356–364.

[11] Kang, J. Y., and Cochran, J. E., Jr., "Stability Criteria of Slosh Motion with Periodicity in a Spinning Spacecraft," *Journal of Guidance, Control, and Dynamics*, Vol. 28, No. 3, May 2005, pp. 562–567.

[12] Hughes, P. C., *Spacecraft Attitude Dynamics*, Wiley, New York, 1986, p. 526.